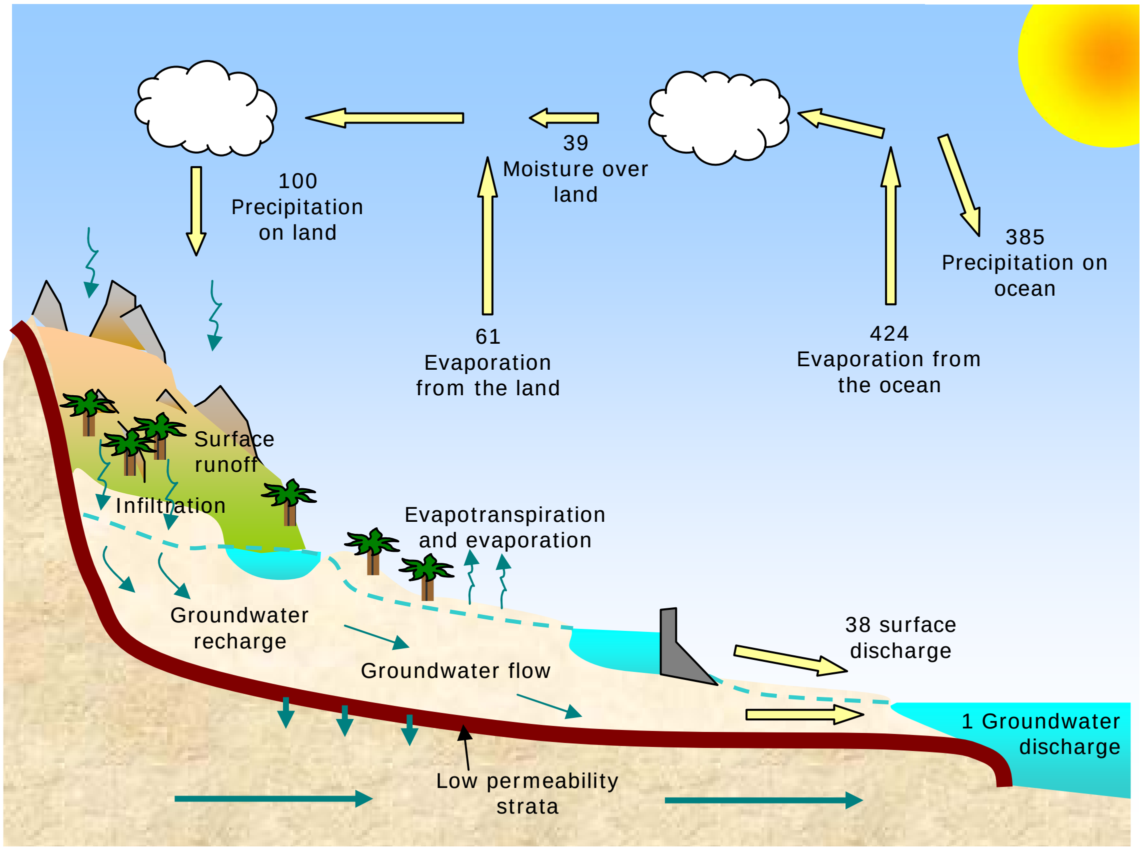
1.72, Groundwater Hydrology
Prof. Charles Harvey

# Lecture Packet #1: Course Introduction, Water Balance Equation

## Where does water come from?

It cycles. The total supply doesn't change much.



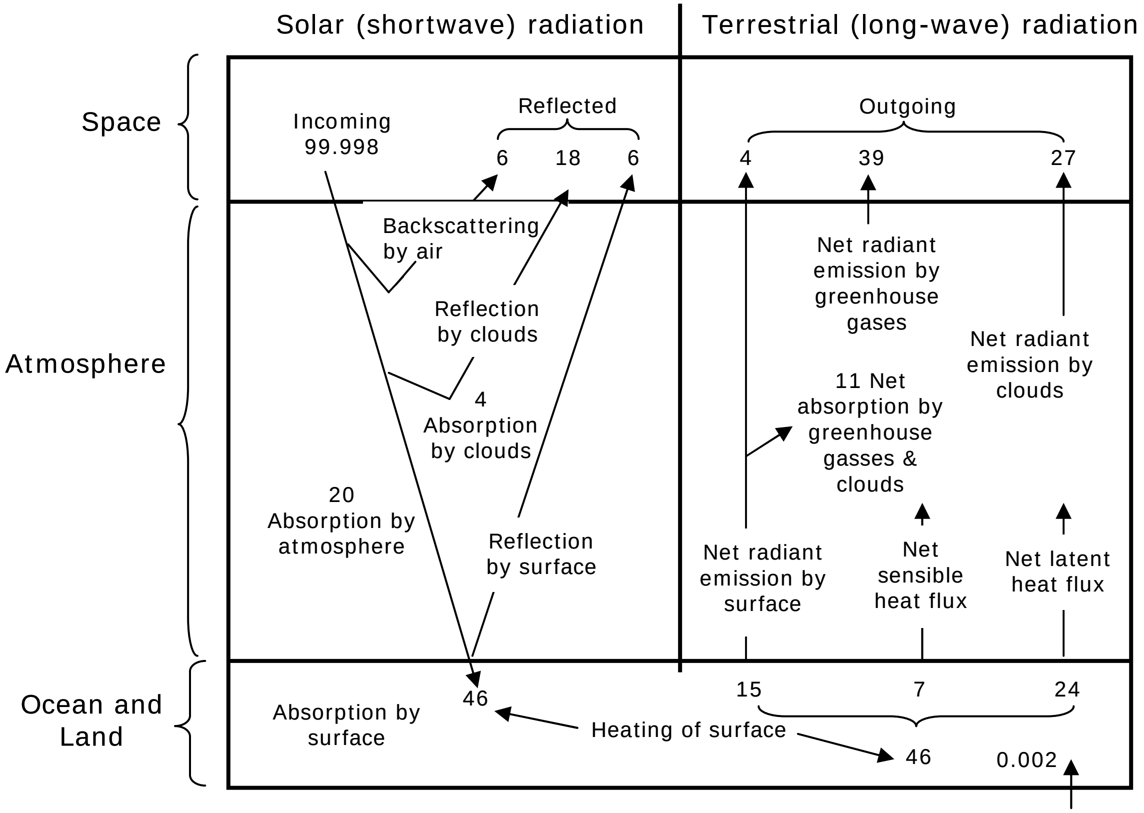
**Hydrologic cycle with yearly flow volumes based on annual surface precipitation on earth, ~119,000 km$^3$/year.**

<u>3000 BC – Ecclesiastes 1:7 (Solomon)</u>
"All the rivers run into the sea; yet the sea is not full; unto the place from whence the rivers come, thither they return again."

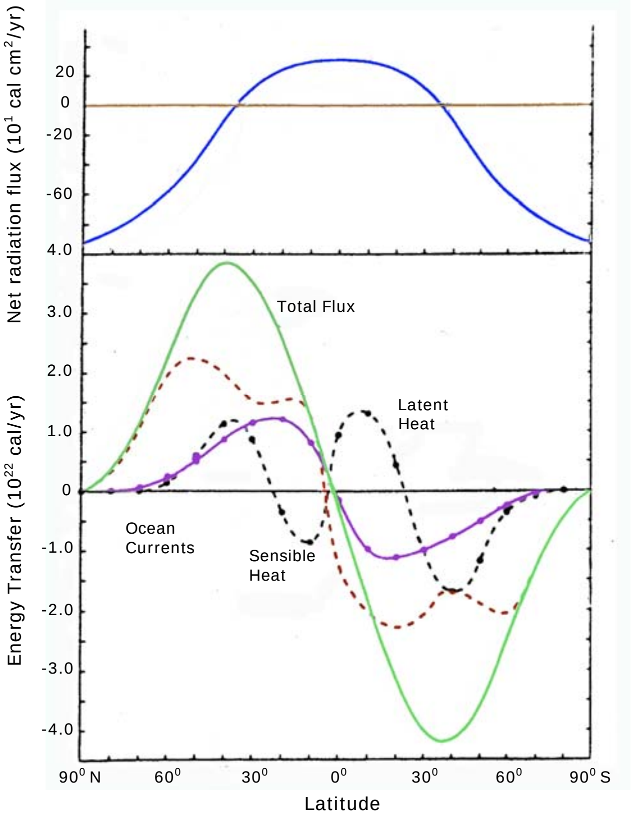
<u>Greek Philosophers (Plato, Aristotle)</u> embraced the concept, but mechanisms were not understood.

<u>17th Century</u> **–** Pierre Perrault showed that rainfall was sufficient to explain flow of the Seine.

## The earth's energy (radiation) cycle

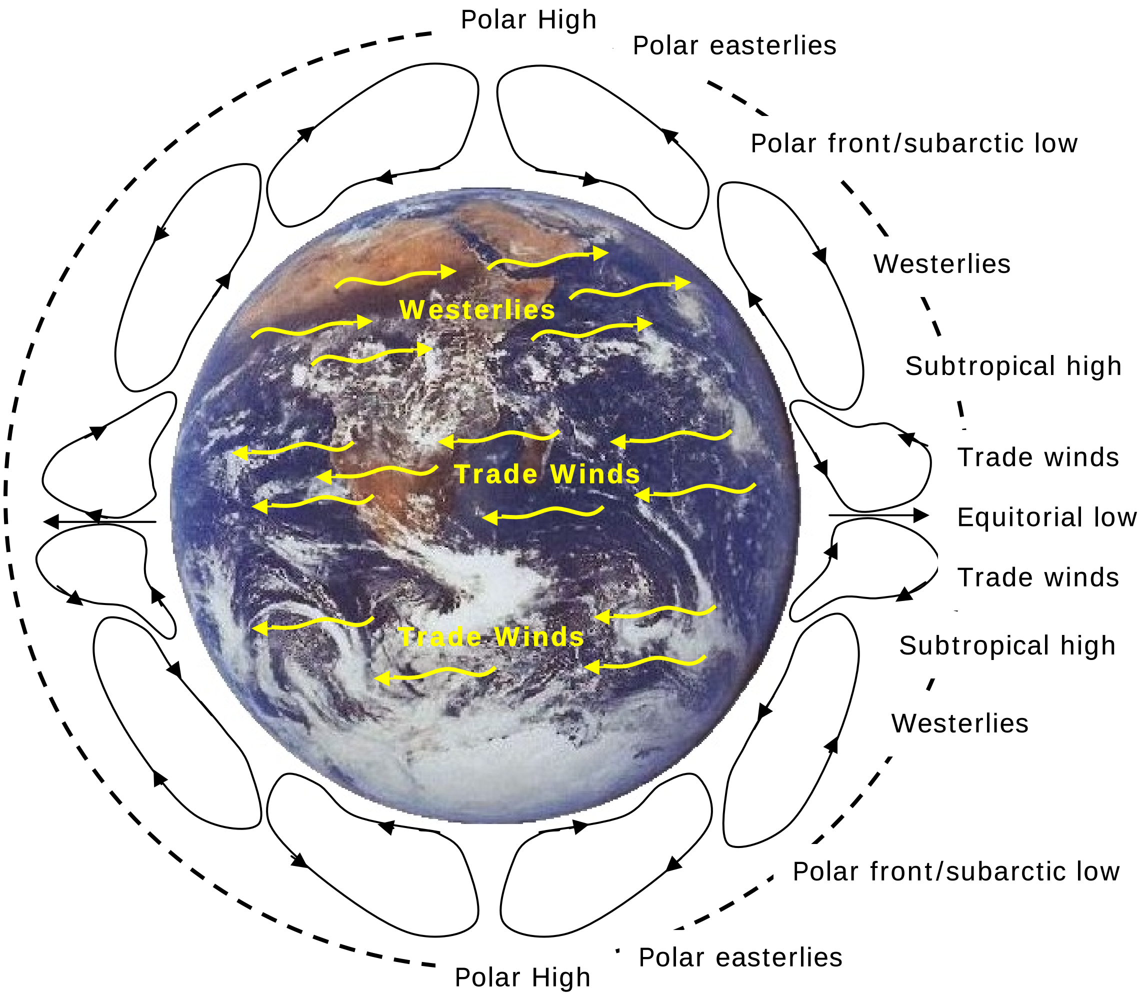


## Circulation redistributes energy

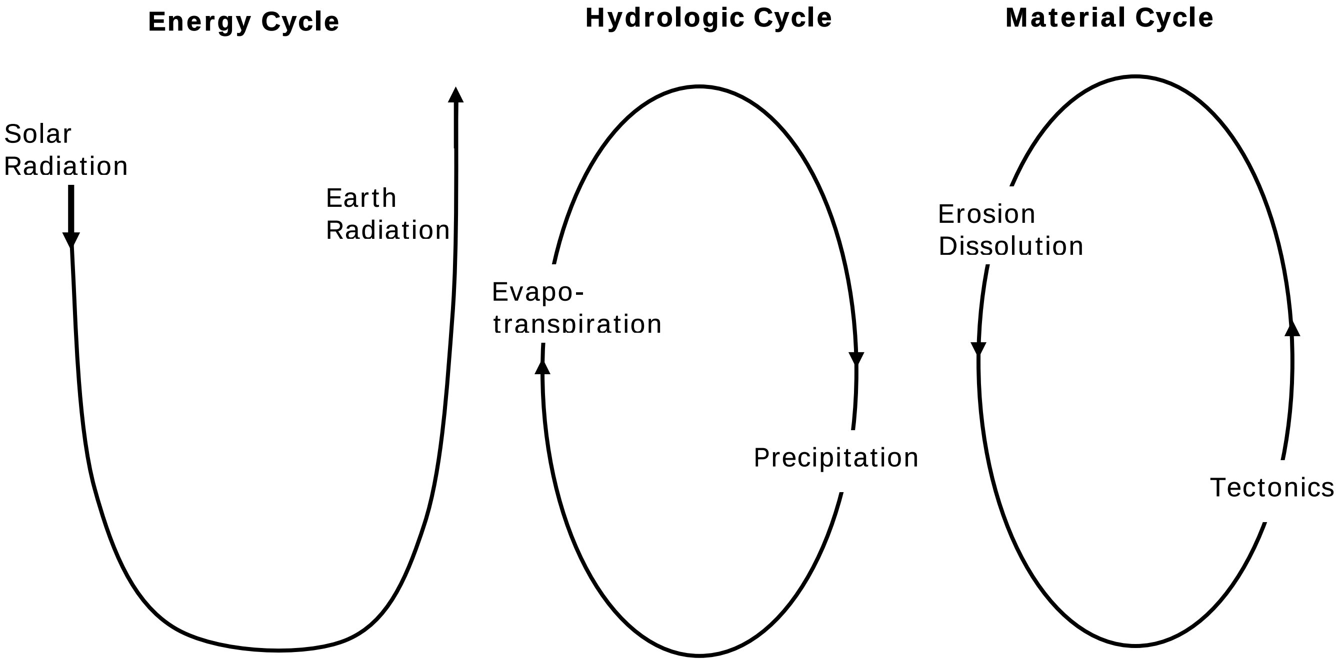


## The Earth's Energy (Radiation) Cycle:

**Spatial distribution of energy and temperature drives circulation – both global and local:**



## Coupled Earth Cycles

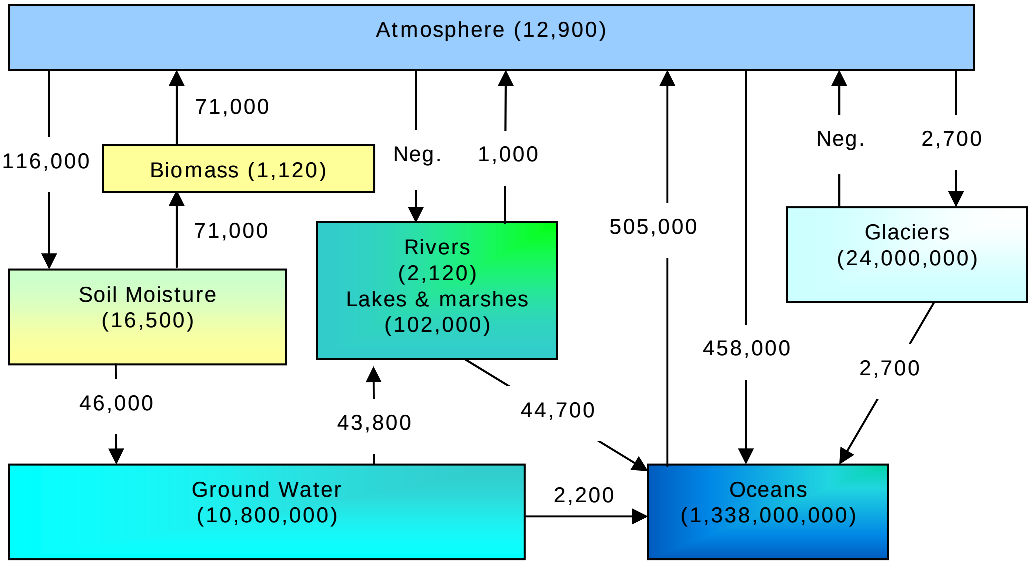


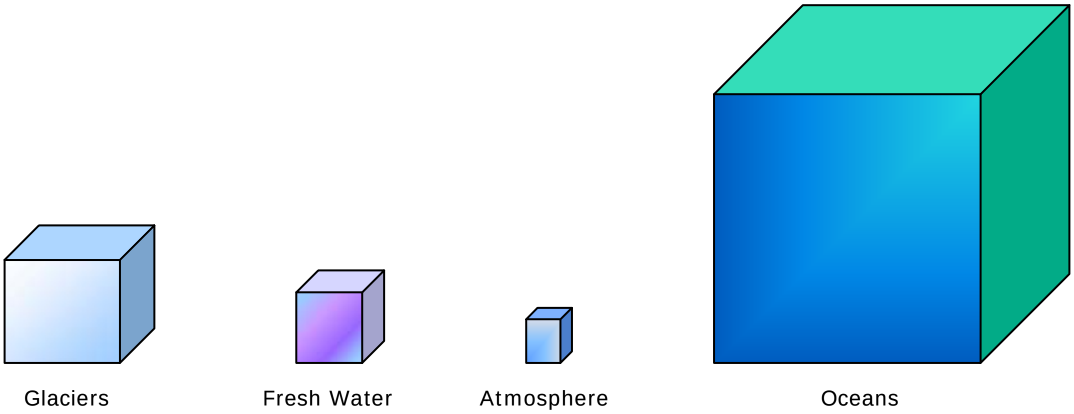
Estimates from river outflows indicate 17 x $10^9$ Tons/Year of material is transported into the Ocean. Another 2 or 3 x $10^9$ Tons/Year is trapped in reservoirs.

80% of material transported is particulate
20% is dissolved

0.05 mm/yr (may have been accelerated by man, and actual rates may be much larger)

Local rates depend on relief, precipitation, and rock type.





**Relative volumes of water in glaciers, fresh water, atmosphere and oceans.**

## Estimate of the World Water Balance

| Parameter | Surface area (km$^2$) X 10$^6$ | Volume (km3) X 10$^6$ | Volume % | Equivalent depth (m) | Residence Time |
|---|---|---|---|---|---|
| Oceans and seas | 361 | 1370 | 94 | 2500 | ~4000 years |
| Lakes and reservoirs | 1.55 | 0.13 | <0.01 | 0.25 | ~10 years |
| Swamps | <0.1 | <0.01 | <0.01 | 0.007 | 1-10 years |
| River channels | <0.1 | <0.01 | <0.01 | 0.003 | ~2 weeks |
| Soil moisture | 130 | 0.07 | <0.01 | 0.13 | 2 weeks – 1 year |
| Groundwater | 130 | 60 | 4 | 120 | 2 weeks – 10,000 years |
| Icecaps and glaciers | 17.8 | 30 | 2 | 60 | 10-1000 years |
| Atmospheric water | 504 | 0.01 | <0.01 | 0.025 | ~10 days |
| Biospheric water | <0.1 | <0.01 | <0.01 | 0.001 | ~1 week |

Amazon is 6,000 km$^3$/yr (~5x more than Zaire-Congo)
0.0003% is potable and available.

What are water needs for humans?

Primitive conditions – 3 to 5 gallons/day
Urban use – 150 gallons/day
US Fresh Water Use – 1,340 gallons/day

Where does the water go?  To make things…to clean things….

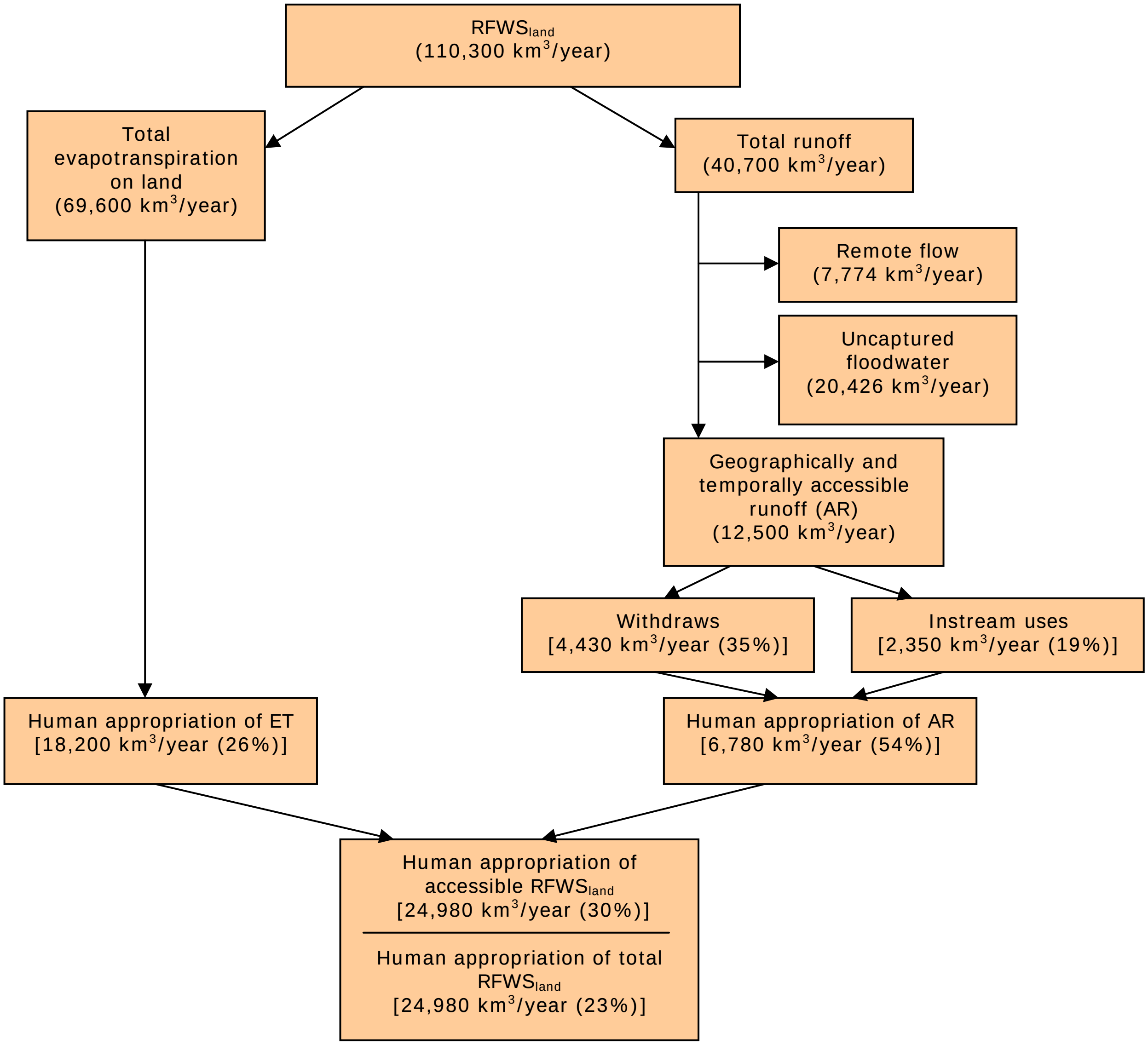
| **Item** | **Gallons** |
|---|---|
| 1 pound of cotton | 2,000 |
| 1 pound of grain-fed beef | 800 |
| 1 loaf of bread | 150 |
| 1 car | 100,000 |
| 1 kilowatt of electricity | 80 |
| 1 pound of rubber | 100 |
| 1 pound of steel | 25 |
| 1 gallon of gasoline | 10 |
| 1 load of laundry | 60 |
| 1 ten-minute shower | 25-50 |

**Some points:**

- A lot of water is used for agriculture
    - 56% (76 BGD) is consumptive use
    - 20% (28 BGD) is lost in conveyance
    - 24% (33 BGD) is return flow
- A lot of water is used for thermoelectric power generation
    - 87% of all industrial water use
- A small savings in either of these categories would free up significant quantities for public supplies

**Water Use in the US in 1990**

| **Fresh Water Use 1990<br>Total 339 BGD** | |
|---|---|
| California | 35.1 |
| Texas | 20.1 |
| Idaho | 19.7 |
| Illinois | 18.0 |
| Colorado | 12.7 |
| Louisiana | 11.7 |
| Michigan | 11.6 |
| New York | 10.5 |
| Pennsylvania | 9.8 |
| Indiana | 9.4 |
| Montana | 9.3 |
| Four states account for 27% | |
| 11 states account for 50% | |



Analysis of human appropriation of renewable freshwater supply (RFSW) on land. *Figure adapted from Postel et al., Science, (271) p. 758, Feb. 9, 1996*

## Postel et al.'s Calculations

1) Calculation of appropriated ET – indirect
   - 132 billion tons of biomass produced a year (Vitousek, 1992).
   - 30% used by people.
   - Approximate people's use of ET from the proportion of biomass – 30%.
   - Subtract agricultural irrigation (2,000 $km^3$/year) and assume half the water. from parks and lawns is irrigation (80 $km^3$/year).

2) Inaccessible runoff
   - 95 of the Amazon, Half of the Zaire-Congo and all of the Polar rivers 7774 .$km^3$/year (did not consider other northern rivers – conservative?).
   - Flood water. 11,100 $km^3$/year of runoff is base flow (6).
   - Capacity of man-made reservoirs is 5,500 $km^3$/year.
   - Accessible runoff = Baseflow + reservoir capacity = 12,500.

$\nwarrow$ adjusted for accessible

3) Withdrawals (consumed)
- 12,000 $m^3$/ha average irrigation water application.
- 240,000,000 ha of world irrigated area.
- 2,880 $km^3$/year.
- 65% consumed = 1,870 $km^3$/year.

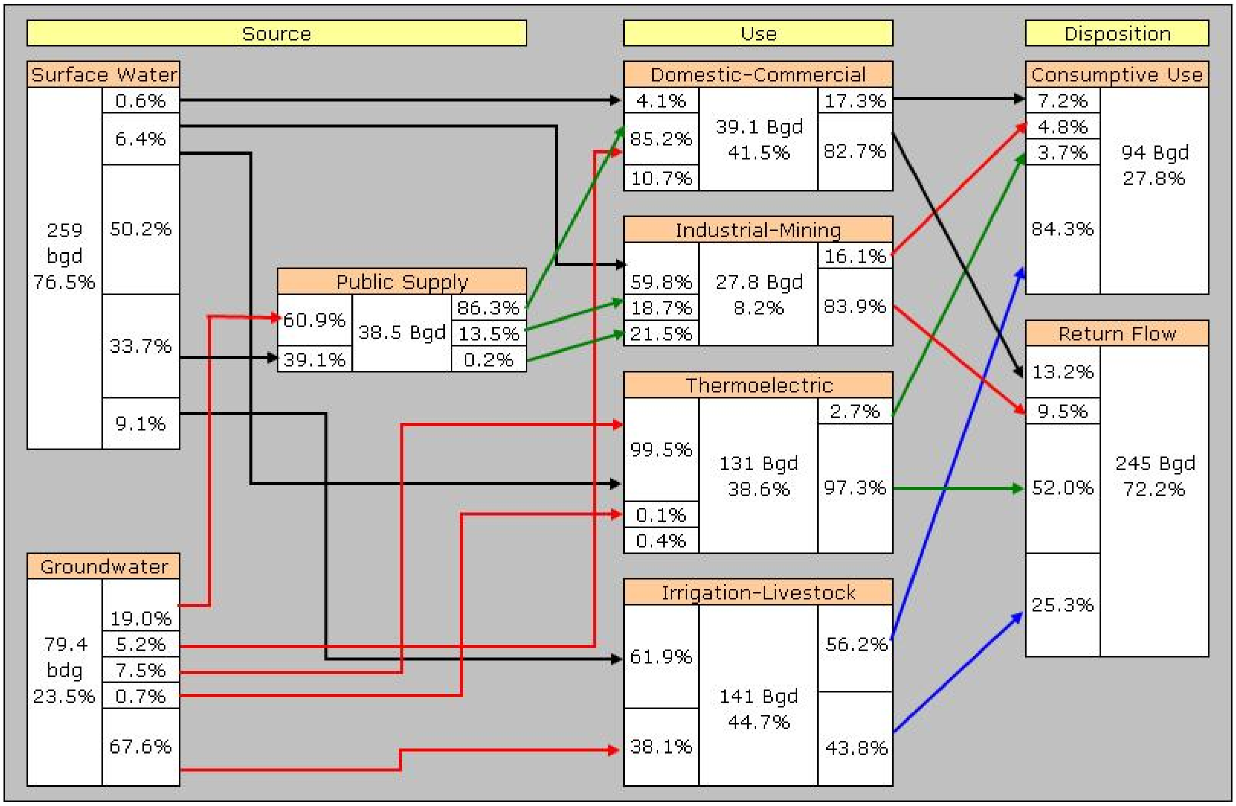
- Industrial use: 975 $km^3$/year, 9% consumed, 90 $km^3$/year.

- Municipal use: 300 $km^3$/year, 17% consumed, 50 $km^3$/year.

- Reservoir losses – 5% loss, 275 $km^3$/year.

- Total consumed = 2,285 $km^3$/year.

4) Instream use
- 28.3 L/s per 1,000 population, applied to 1990 population, 4,700 $km^3$/year.
- Assuming 50% of waste gets treatment – 2,350 $km^3$/year.
- Neglects dispersed pollution (agricultural) and flood waters.



Estimated water use in the United States, billion gallons per day (bgd), for domestic and commercial purposes. *Adapted from Solley, Pierce, and Perlman, 1993.*

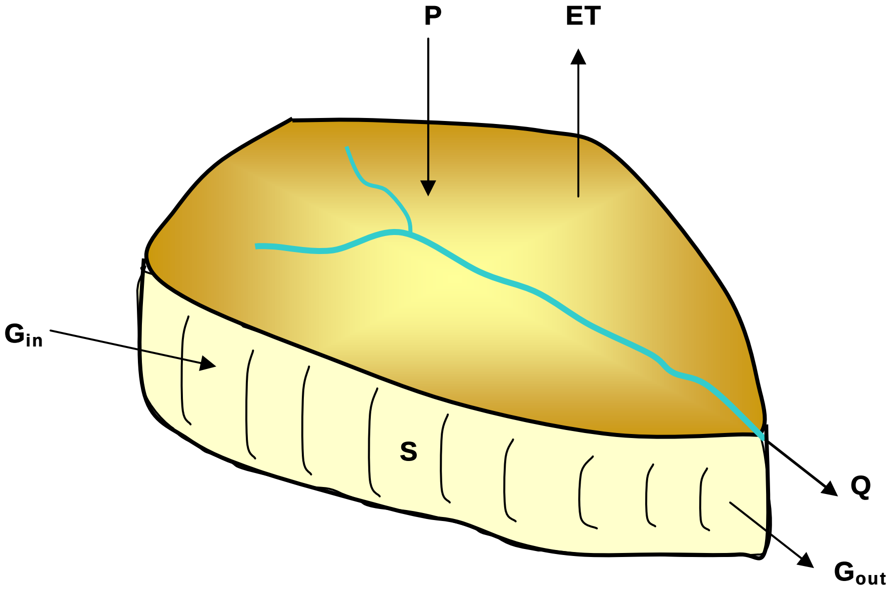
## Relative Merits of Surface and Subsurface Reservoirs

| Surface Reservoirs | Subsurface Reservoirs |
|---|---|
| Disadvantages | Advantages |
| Few new sites free (in USA) | Many large-capacity sites available |
| High evaporative loss, even where humid climate prevails | Practically no evaporative loss |
| Need large areas of land | Need very small areas of land |
| May fail catastrophically | Practically no danger of failure |
| Varying water temperature | Water temperature uniform |
| Easily polluted | Usually high biological purity, although pollution can occur |
| Easily contaminated by radioactive fallout | Not rapidly contaminated by radioactive fallout |
| Water must be conveyed | Act as conveyance systems, thus obviating the need for pipes or canals |

## Watershed Hydrologic Budgets

Delineation of a watershed (drainage basin, river basin, catchment)

- Area that topographically appears to contribute all the water that flows through a given cross section of a stream. In other words, the area over which water flowing along the surface will eventually reach the stream, upstream of the cross-section.
- Horizontal projection of this area is the drainage area.
- The boundaries of a watershed are called a divide, and can be traced on a topographic map by starting at the location of the stream cross-section then drawing a line away from the stream that intersects all contour lines at right angles. If you do this right, the lines drawn from both sides of the stream should intersect. Moving to either side



**Water Balance Equation**

$$\frac{\partial S}{\partial t} = P + G_{in} - \left(Q + ET + G_{out}\right)$$

At steady-state: $\frac{\partial S}{\partial t} = 0$

$$P + G_{in} = \left(Q + ET + G_{out}\right)$$

Hydrologic Production

Runoff, or Hydrologic Circulation

Q and P are the only quantities that we can try to measure directly. If steady state is assumed, these measurements can be used to calibrate models of evapotranspiration and groundwater flow.

From and engineering point of view we are interested in understanding what controls Q.

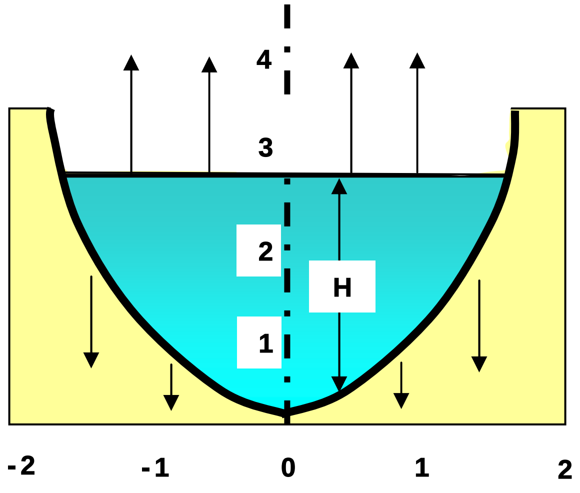
$$Q = P + (G_{in} - G_{out}) - ET$$

How much of Q is available to human use?

How can we increase Q?

**Lake Parabola**

(A simple quadratic model of the cross section of a circular lake)



$$H = cr^2$$

$$A = \pi r^2 = \frac{\pi}{c} H$$

$$V(H) = \int_0^H A(h)dh = \int_0^H \frac{\pi}{c} H dh = \frac{\pi}{2c} H^2$$

## Discharge into Lake Parabola

If there is constant volume flux Q $m^3$/day into the lake, how does the depth depend on time? (At time 0, the depth is $H_o$ m)

Since there is no outflow, the charge in storage must equal the inflow

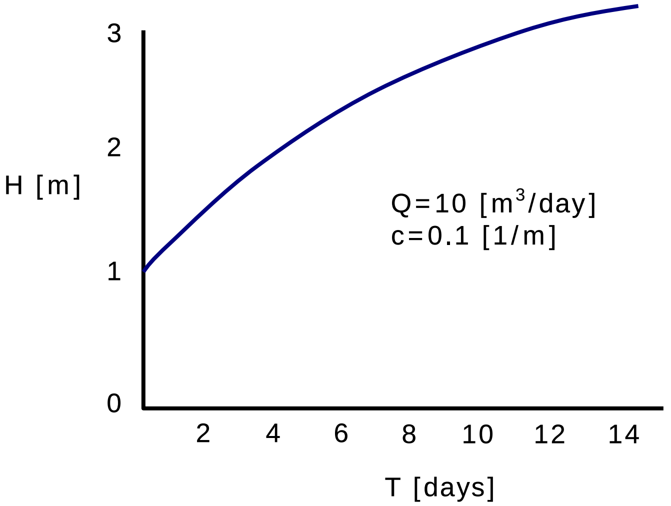
Mass-balance (water balance) equation

$$Q = \frac{dV(H)}{dt} = \frac{\pi}{2c}\frac{d(H^2)}{dt} = \frac{\pi}{c} H \frac{dH}{dt}$$

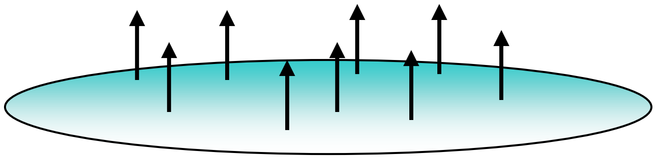
This differential equation can be solved by separation of variables

$$\int_0^T \frac{cQ}{\pi} dt = \int_{H_o}^{H} h dh$$

$$\frac{cQ}{\pi} T = \frac{H^2}{2} - \frac{H_o^{\,2}}{2} \qquad H = \sqrt{\frac{2cQ}{\pi} T + H_o^{\,2}}$$



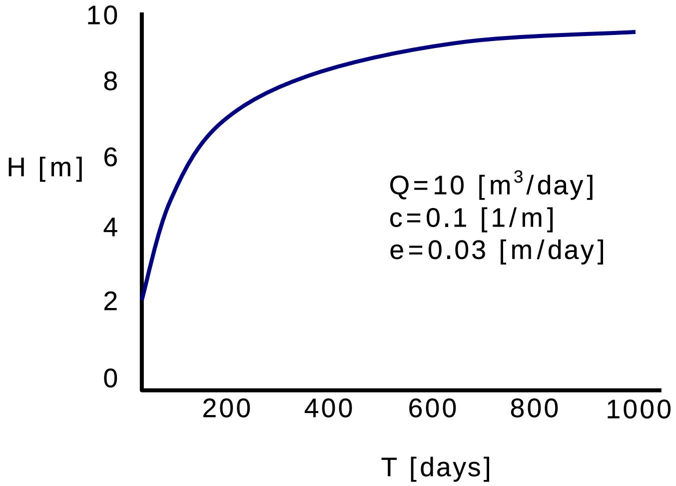
## Discharge into and Evaporation out of Lake Parabola



e = rate of evaporation [L/T]

$$\frac{dV(H)}{dt} = Q - Ae = Q - \frac{\pi}{c} He$$

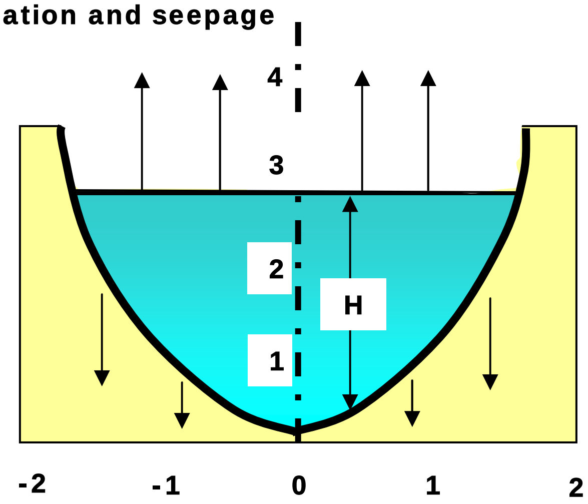
$$\frac{\pi}{c} H \frac{dH}{dt} = Q - \frac{\pi}{c} He \qquad\qquad \frac{dH}{dt} = \frac{Qc}{\pi H} - e$$



**Steady State Solution** $$\cancel{\frac{dH}{dt}} = \frac{Qc}{\pi H} - e$$

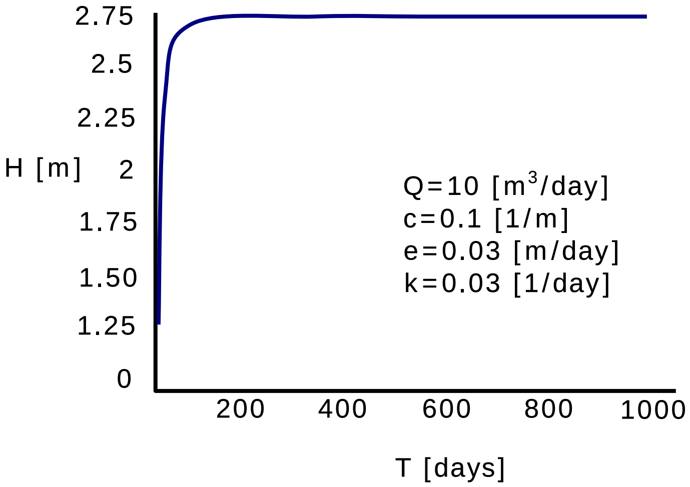
$$H = \frac{Qc}{\pi e} = \frac{(10)(0.1)}{\pi(0.03)} = 10.6 \ \boldsymbol{m}$$

**Discharge, evaporation and seepage**



$$\frac{dV(H)}{dt} = Q - eA - kHA = Q - \frac{e\pi}{c}H - \frac{k\pi}{c}H^2$$

$$\frac{\pi}{c}H\frac{dh}{dt} = Q - \frac{e\pi}{c}H - \frac{k\pi}{c}H^2 \qquad\qquad \frac{dh}{dt} = \frac{Qc}{\pi H} - e - kH$$



**Steady State Solution**

$$\cancel{\frac{dh}{dt}} = \frac{Qc}{\pi H} - e - kH$$

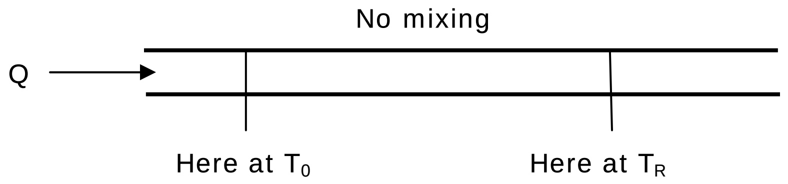
$$\frac{Qc}{\pi} - eH - kH^2 = 0$$

Quadratic formula:

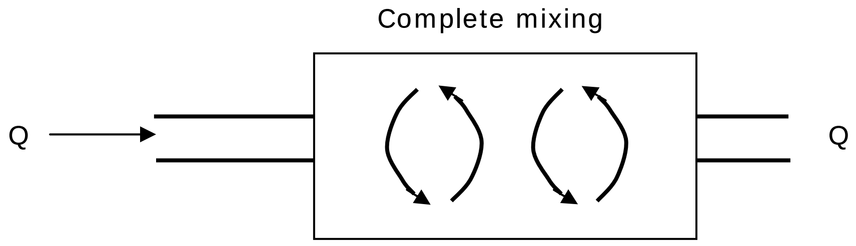
$$H = \frac{-e\pi \pm \sqrt{(e\pi)^2 - 4ck\pi Q}}{2k\pi} = \textit{-3.8 m, 2.8 m}$$

### Residence Time (steady state, complete mixing)

Very useful concept for degradation and chemical reactions



$V = QT_R$ $T_R = V/Q$ Exactly



$T_R = V/Q$ On Average

**For this problem:**

Consider discharge and evaporation out of Lake Parabola. Assume no seepage.

$Q$ = input = output = 10 [$m^3$/day]

$$V(H) = \frac{\pi}{2c}H^2 = \frac{\pi}{2(0.1)}10.6^2 = 1{,}765m^3$$

$$T_R = V/Q = \frac{1{,}765m^3}{10m^3/day} = 176.5days$$

Consider yourself:

| Quantity | Men | Women |
|---|---|---|
| Water weight [Kg] | 60 | 50 |
| Average Intake [Kg/day] | 3 | 2.1 |
| Residence Time [day] | 14 | 14 |