14.54 International Economics
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Problem Set 2
(Due on 10/4)

1. Specific factor model

In this economy two countries, Home and Foreign, produce two goods, Computers and Desks, using three factors of production, skilled labor (H), unskilled labor (L) and capital (K).

Skilled workers are self-employed. They rent capital at the rental rate $r$ and produce computers according to the production function:

$$Q_C = H^{1/2}K_C^{1/2},$$

their income is given by the profits $R_C = P_C Q_C - r K_C$.

Unskilled workers are also self-employed. They rent capital and use it to produce desks according to the production function:

$$Q_D = L^{1/2}K_D^{1/2},$$

their income is given by the profits $R_D = P_D Q_D - r K_D$.

The Home country is endowed with the following factor amounts: $H = 9, K = 20, L = 4$.

The Foreign country is endowed with the following factors: $H^* = 1, K^* = 20, L^* = 4$.

Preferences are identical in the two countries and are described by the following utility function:

$$U(C_C, C_D) = C_C^{1/3} C_D^{2/3}$$

Remember that in the economy all the factors are always fully employed.

Assume for now that the two countries are in autarky.

1. Derive the production possibility frontier for both countries using the 4-quadrant graph seen in class.

2. Given prices of Computers and Desks, $p_C$ and $p_D$, consider the Home country and find the autarky equilibrium return to capital $r_K$ using the graph showing the Value Marginal Product of Capital (p × MPK) for both goods. What is the allocation of Capital to the production of the two goods?

3. Now consider the Foreign country and, for given prices $p_C^*$ and $p_D^*$ find the return to capital $r_K^*$ using the same method as in part 2. What is the allocation of Capital to the production of the two goods?

4. Take the Home country and consider an increase in the relative price $\frac{p_C}{p_D}$. What is the
5. From parts 2 and 3 you are able to derive the relative supply of the two goods in both countries. Knowing preferences you can also derive the relative demand for each country. Using relative demand and relative supply find for each country the equilibrium autarky relative price: \( \left( \frac{p_C}{p_D} \right)_A \) and \( \left( \frac{p_C}{p_D} \right)_A^* \). You should use Desk as the numeraire good by setting its price to one.

6. Now imagine the two countries can trade. Derive the world relative supply curve and find the equilibrium world relative price. Now using your answer to part 4 show which factors gain and which lose from allowing free trade between the two countries.

2. From Autarky to Trade with Transfers (in Specific Factors Model)

1. Calculate the indirect utility (function of prices and income) for each agent in the home country under the autarky equilibrium considered above. You should still consider desk as the numeraire good, such that \( p_D = 1 \).

2. Now calculate the indirect utility (function of prices and income) for the Home country under the trade equilibrium considered above. To simplify the calculations, assume that the price of desks, the numeraire good, is \( p_D = 2^{-1/6} \).

3. Calculate the total gains from trade in the Home country.

4. Assume that, in the home country, a social planner can enforce a transfer scheme such that all the gains from trade go to the unskilled worker. The planner can either impose only lump sum taxes or subsidies in order to implement such scheme. Find how much the skilled workers, the capital owners, and the unskilled workers would have to pay/receive in this world (Hint: remember that the sum of all taxes and subsidies must be zero).

5. Now assume that same social planner decides to divide the gains from trade equally among each type of individual in this country. Find how much the skilled workers, the capital owners, and the unskilled workers would have to pay/receive in this world.

3. The Transfer Problem

Suppose that Home (e.g. US) and Foreign (e.g. Germany) countries were at war for the past years and that Home has just won the war. Now, the two countries are fighting over war reparations. The home country argues that Foreign should pay an amount \( T \) to him and that there won’t be any other costs to the Foreign economy. However, Foreign argues that there would be other costs in the form of a terms of trade effect. In order to find out who is right and who is wrong, let’s solve the following problem:

Preferences for the Home country are

\[
a \log x_C + (1 - a) \log x_F
\]

Preferences for the Foreign country are

\[
(1 - a) \log x_C^* + a \log x_F^*
\]

The Home income is the value of its endowment plus a transfer \( T \). And the foreign income
is the value of their endowment plus a transfer $T^*$ (which could be negative). The endowments of each good are the following:

$$e_C = 1, e_F = 0$$
$$e^*_C = 0, e^*_F = 1$$

Assume that

$$\alpha = \frac{1}{2}$$

You should use the good C as the numeraire good by assuming that $p_C = 1$.

1. First, assume that there are no transfers, so that $T = T^* = 0$. Calculate the new terms of trade, the relative world demand and relative world supply in the free trade equilibrium. Draw a supply and demand graph and show this equilibrium.

2. Now suppose the Foreign country transfer $T^* = 0.25$ to the Home country. Calculate the new relative world demand, and relative world supply after the transfer. Is there any effect on the terms of trade? Is the Home country better off? If this was the case, who would have won the argument above?

   Now assume that

   $$\alpha = \frac{2}{3}$$

3. Again, suppose the Foreign country transfer $T^* = 0.25$ to the Home country. Calculate the new terms of trade, relative world demand, and relative world supply after the transfer. Is there any effect on the terms of trade? Is Home better off? Who would have won the argument above?

4. Would it be possible for the Home country to be worse off and the Foreign country to be better off after the transfer? Explain (you should mention the income effects).

**Trade policy issues**

Comment the following excerpts from articles in the New York Times and The Economist in light of the models we have seen in class so far.

1. “The rich worlds agricultural policies deserve to be put to the sword. America, for example, spends almost $4$ billion each year on cotton subsidies alone. This government largesse has turned America, an expensive cotton producer, into a leading cotton exporter. The subsidies are not just a burden on American taxpayers; they also helped to halve world cotton prices between 1997 and 2002, according to the Financial Times. One of the distinguishing features of being poor is being dependent on one or two exports. Burkina Faso and Mali count on cotton for about one-third of their export earnings, and Chad
about one-quarter. The collapse in cotton prices has hit their farmers particularly hard.” [The Economist, September 12th, 2003]

2. "Mr. Vachier, however, said, “Why should world rules dictate whether French taxpayers can pay me to preserve our way of life and protect our countryside?” [New York Times, September 9th, 2003]

3. "This American emphasis on exports has not only increased the size of the farms, but has also limited what is grown on them.[...] Now the commercial crops are down to four” and Gaylord Moeller complains “The subsidies let the big farms get bigger...” [New York Times, September 9th, 2003]