Exercise 1. Give a physical explanation of the meaning of Equation 2.18 (in volume 2 of the text) without the use of equations.

Exercise 2. Let the function \( f(z, t) \) represent a solution to the wave equation. This solution is shown in the following figure as a function of time \( t \) at the position \( z = 0 \).

![Wave Function Graph]

Notice that \( f(0, t) \) is non-zero for \( 0 < t < 2 \) ms and zero elsewhere.

a) Suppose that the wave is propagating in the \(+z\)-direction with a propagation velocity of 100 mm/ms. Plot \( f(z, t) \) versus \( z \) at time \( t = 2 \) ms.

b) Suppose that the wave is propagating in the \(-z\)-direction at a propagation velocity of 100 mm/ms. Plot \( f(z, t) \) versus \( z \) at time \( t = 2 \) ms.

Exercise 3. A 100 \( \mu \)m radius electrically large, cylindrical cell has an internal longitudinal current that is constant in time with a spatial dependence of

\[
I_t(z) = -e^{0z}, \quad \text{for } z < 0
\]
where \( I_s(z) \) has units of \( \mu A \) and \( z \) has units of cm. There are no externally applied currents for \( z < 0 \).

a) Determine the longitudinal current density in the cytoplasm \( J_s(z) \).

b) Determine the current per unit length through the membrane \( K_m(z) \).

c) Determine the current density through the membrane \( J_m(z) \).

d) Determine the total current flowing through the membrane, \( I_m \), in the segment \( -1 < z < 0 \).

**Problem 1.** The membrane of a cell is known to support passive electrodiffusion and active transport of both sodium and potassium. Other ions, solutes, and water are known to be impermeant. The sodium and potassium conductances are \( G_Na \) and \( G_K \), respectively. The pump runs at a constant rate of \( \gamma \) cycles per second, where \( \gamma > 0 \). The density of pumps is \( \Omega \) mol/cm². On each cycle, three sodium ions are transported from the inside of the cell to the outside and two potassium ions are transported from outside the cell to the inside.

The cell is placed in a bath that contains potassium and sodium ions with concentrations \( c^0_K \) and \( c^0_Na \), respectively. The volume of the bath is much larger than the volume of the cell. The cell is allowed to come to quasi-equilibrium, and the resulting intracellular concentrations of potassium and sodium are \( c^i_K \) and \( c^i_Na \), respectively. (Recall that quasi-equilibrium is the condition that will persist forever given that the pumps continue to run forever.)

\[ a) \text{ After coming to quasi-equilibrium, will the cell also be in passive electrodiffusive equilibrium (passive electrodiffusive current for each species is zero)? Explain.} \]

\[ b) \text{ After coming to quasi-equilibrium, will the cell be at rest (total ionic current through membrane equals zero)? Explain.} \]

\[ c) \text{ Determine an expression for } c^i_K \text{ in terms of } c^0_K, c^0_Na, c^i_Na, G_K, G_Na, \gamma, \Omega, \text{ and physical constants. [Hint: It may be helpful to first develop a relation between the Nernst equilibrium potentials for sodium and potassium.]} \]

\[ d) \text{ Ouabain is now added to the bath to block the ion pumps. Within seconds after the pumps are blocked, the cell comes to rest. Will the new resting potential be greater than, smaller than, or equal to the resting potential just before the addition of ouabain? Explain.} \]

\[ e) \text{ After coming to rest in ouabain, will the cell also be in passive electrodiffusive equilibrium? Explain.} \]

**Problem 2.** Consider the model of a cell shown in Figure 1. The cell has channels for the passive transport of sodium, potassium, and chloride as well as a pump that actively transports sodium out of the cell and potassium into the cell. The pump ratio is \( I^0_Na/I^0_K = -1.5 \). The following table shows the intracellular and extracellular concentrations, Nernst equilibrium potentials, and conductance ratios for sodium and potassium. Some information is also given for chloride; blank entries represent unknown quantities.

<table>
<thead>
<tr>
<th>( c^0_n ) (mmol/L)</th>
<th>( c^i_n ) (mV)</th>
<th>( G_n/G_K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na⁺ 10</td>
<td>140</td>
<td>+68</td>
</tr>
<tr>
<td>K⁺ 140</td>
<td>10</td>
<td>−68</td>
</tr>
<tr>
<td>Cl⁻ 150</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The cell also contains impermeant intracellular ions. Assume that the cell is in equilibrium at $t = 0$, i.e., assume that at $t = 0$ the cell has reached a condition for which all solute concentrations, the cell volume, and the membrane potential are constant.

a) Choose one of the following statements and explain why it is true.

i) The cell resting potential depends on $G_{Cl}$.

ii) The cell resting potential depends on $V_{Cl}$.

iii) The cell resting potential depends on both $G_{Cl}$ and $V_{Cl}$.

iv) The cell resting potential does not depend on $G_{Cl}$.

v) The cell resting potential does not depend on $V_{Cl}$.

vi) The cell resting potential does not depend on either $G_{Cl}$ or $V_{Cl}$.

b) Determine $V_m^0$.

c) At $t = 0$, the external concentration of chloride is reduced from 150 mmol/L to 50 mmol/L by substituting an isosmotic quantity of an impermeant anion for chloride. Assume that the concentrations of sodium and potassium both inside and outside the cell remain the same and that the volume of the cell does not change.

i) Determine $V_m^0(0^+)$, the value of the membrane potential immediately after the change in solution. You may ignore the effect of the membrane capacitance.

ii) Determine $V_m^0(\infty)$, the value of the membrane potential after the cell has equilibrated.

iii) Determine $c_{Cl}^i(\infty)$, the intracellular chloride concentration after the cell has equilibrated.

iv) Give a physical interpretation of your results in i), ii), and iii).

v) Discuss the validity of the assumptions that the sodium and potassium concentrations in the cell are constant and that the volume does not change.
Problem 3. A uniform, isolated, small cell has a membrane that is permeable to sodium and potassium ions only and contains an active transport mechanism that transports 3 sodium ions outward and 2 potassium ions inward for every molecule of ATP split into ADP and phosphate. Summed over the entire membrane of this cell, the active transport system splits $10^{-17}$ moles of ATP per second. Assume that the cell is at quasi-equilibrium so that the concentrations of all ions are constant. The cell has a total membrane conductance of $10^{-10}$ siemens. The temperature is 24°C. The ionic concentrations of sodium and potassium across the membrane are given in the following table.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Concentration (mmol/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Internal</td>
</tr>
<tr>
<td>Sodium</td>
<td>15</td>
</tr>
<tr>
<td>Potassium</td>
<td>150</td>
</tr>
</tbody>
</table>

The potassium conductance exceeds the sodium conductance of this cell.

a) Determine the value of the component of the resting membrane potential, $V_m^0$, that is attributable directly to active transport.

b) Determine the value of the resting membrane potential, $V_m^0$.

c) Determine the values of the sodium, $G_{Na}$, and potassium, $G_K$, conductances of the membrane.

Problem 4. The following two experiments are performed on a squid giant axon:

- **Experiment #1**: The axon is placed in a large volume of sea water, and the size of the transmembrane action potential is measured by means of an intracellular micropipet and is found to have a peak-to-peak value of 100 mV. The conduction velocity is 36 m/s.

- **Experiment #2**: The axon is placed in oil and the trans-membrane potential is still found to be 100 mV peak-to-peak. The peak-to-peak size of the extracellular action potential is 75 mV.

Estimate the expected conduction velocity in Experiment #2. State your assumptions.
**Problem 5.** A fine platinum wire with a resistance per unit length of 130 Ω/cm is inserted inside a portion of a squid axon as illustrated below.

The wire is so thin that its volume can be ignored. The axon (500 μm diameter) is electrically stimulated to produce a propagated action potential traveling in the +z direction. The action potential is recorded at two intracellular sites: \( V_1(t) \) is recorded at \( z = z_1 \) and \( V_2(t) \) is recorded at \( z = z_2 \). The distance between the stimulus electrode (not shown) and \( z_1 \) is 2 cm. Results are shown in the following figure.

The resistivity of the axoplasm of this axon is 23 Ω·cm. The resistance per unit length of the external solution is 1.2 Ω/cm. The wire begins at some location between \( z_1 \) and \( z_2 \), but the exact position of the beginning is not known and should not be used in any of your calculations.

a) Determine the instantaneous speed of the action potential as it’s peak passes the point \( z = z_1 \).

b) Determine the instantaneous speed of the action potential as it’s peak passes the point \( z = z_2 \).

c) Sketch the extracellular potential as a function of space \( (z) \) that results at the time that the peak of the action potential passes the point \( z = z_1 \). Include distances \( z_1 - 4 < z < z_1 + 4 \) cm. Indicate the scale for the y axis. Describe the important features of this plot.

d) Sketch the extracellular potential as a function of space \( (z) \) that results at the time that the peak of the action potential passes the point \( z = z_2 \). Include distances \( z_2 - 4 < z < z_2 + 4 \) cm. Indicate the scale for the y axis. Describe the important features of this plot.