Week 2 Review

What was covered:
- Dissolve and diffuse
- 2 compartment model
- thin vs. thick membrane
- measurements in cells

Dissolve and diffuse:
Assume: dissolve is much faster than diffuse
\[ k = \text{partition coefficient (dissolve)} \]
\[ D = \text{diffusivity (diffuse)} \]
\[ \phi = -kD \frac{\partial c}{\partial x} - \frac{\partial \phi}{\partial x} = \frac{\partial c}{\partial t} \]
\[ \frac{\partial c}{\partial t} = kD \frac{\partial^2 c}{\partial x^2} \]

2 compartment model
\[ A= \text{cross-sectional area} \]

Assume:
- well stirred baths (in baths \( c(x,t) = c(t) \))
- solute is conserved (nothing is eating it up or producing it)
- baths big compared to membrane
- thin membrane

Steady State (SS) time constant: \( \tau_{ss} = \frac{d^2}{\pi^2 D} \)
If at SS then:
\[ \phi = \frac{Dk}{d} (c_1(t) - c_2(t)) = P(c_1(t) - c_2(t)) \quad \text{Fick’s law for membranes} \]
P is the membrane permeability
\[ \phi(x,t) = -\frac{1}{A} \cdot \frac{d}{dt} (n(x,t)) \quad \text{where } n \text{ is number of solutes} \quad \text{Definition of flux} \]

From Fick’s law for membranes, can get the equilibrium time constant, \( \tau_{EQ} \):
\[ \tau_{EQ} = \frac{1}{AP \left( \frac{1}{V_1} + \frac{1}{V_2} \right)} \quad \text{(see supplement for derivation)} \]

Thin vs. thick membranes
When does this theory break down?
Compare \( \tau_{SS} \) to \( \tau_{EQ} \): If \( \tau_{EQ} \gg \tau_{SS} \) then thin membrane...
However, if \( \tau_{EQ} \) is on the order of \( \tau_{SS} \) then not thin membrane:
What does this mean:
1. time to get to SS cannot be ignored
2. concentration in baths will change significantly before reaching SS
3. amount of solute in membrane might not be negligible
4. overall time profiles of concentration/flux are NOT exponentials (can’t reduce to Fick’s law for membranes so profiles are not solutions to 1st order linear differential equation)

Measurements:

(To measure time constant of exponential curve: extend a line at initial time and intersecting with the asymptote… see problem set 1)

How to measure $\tau_{SS}$?

On SMALL time scale:
1. look at plot of concentration profile in membrane (remember: on short time scale, only membrane concentration is changing; bath concentrations are not changing significantly at this point.)
2. look at plot of $\phi(t)$

How to measure $\tau_{EQ}$?

On LARGE time scale:
1. look at plots of concentration. (in bath or membrane)
2. look at plot of $\phi(t)$

If you aren’t comfortable with figuring out time constants and stuff like that from concentration and flux plots review problem 4 and 5 on pset #2 and practice with the simulation software… (and if you are still confused, feel free to ask us (the Tas) questions!! 😊)

More measurements:
Be comfortable with the plots Prof. Freeman put up in lecture which kind of look like this:
See pg. 145 in the text for nicer graph:

where $P$ is the permeability of a solute and $k$ is the partitioning coefficient.
1. since linearly P is linearly dependent on partition coefficient (which was measured in oil), membrane is lipid
2. bigger solutes (M larger) diffuse more slowly (plot above assumed $D$ was the same for all solute)
3. if there is a solute that is really off from line (even when you take M into account), probably has specialized transport mechanism in the cell