

Problem Set 1

Let's Make a Deal—You are a contestant in a TV game show. In front of you are three closed doors, behind one of which there is a fabulous prize (and behind the others, nothing). The host asks you to choose one door in the hope of winning the prize behind it. After you make your choice, however, he opens one of the other two doors (which, of course, does not reveal the prize) and asks if you would like to switch your choice to the other of the two remaining closed doors. What should you do?

This is famous problem, commonly arising in job interviews (e.g. at Microsoft or Goldman Sachs), has confused many a professional mathematician. Like many probability questions, even when you think you have the solution, you are often be unsure it is correct. Since it is difficult to perform a real experiment enough times to see the trend, as with other problems in this course, an easier way to become (almost completely) convinced is to perform a *computer experiment*.

1. Write a computer program (in the language of your choice) to determine the probability p that you will win the prize if you switch your choice of door after seeing the host open one of the two you did not select at first. Use a (pseudo-)random number generator to place the prize behind one of three “virtual doors” and then to help the host choose which door to open. Repeat this experiment N times, and let p_N be the fraction of times you win the prize by switching, where N is a large number. Extrapolate your data to determine, $p = \lim_{N \rightarrow \infty} p_N$, and decide whether or not it is advantageous to switch doors ($p > 1/2$).
2. Guess the exact value of p from your data, and try to prove you are correct analytically.
3. Let $\epsilon_N = |p_N - p|$ be the sampling error in your computer simulation. Plot $\log \epsilon_N$ versus $\log N$, and try to fit it with a straight line (using linear regression, or simply by estimating with a ruler). Determine the exponent ν , defined by, $\epsilon_N \propto N^{-\nu}$, which governs the rate of convergence of your numerical estimate.
4. Repeat problems 1 and 2 for the more general situation where the prize is behind one of n doors, and, after your initial choice, the host opens m of the $n - 1$ doors you did not choose (not containing the prize) and then offers you a chance to switch your choice.